Calculation of Binding Energies, One and Two Nucleon Separation Energies, and Q_α Values for All Nuclei

Erik Olsen

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The following illustrates how calculations are made on MassExplorer for the binding energies of odd-A and odd-odd nuclei and the separation energies and Q_α values of all nuclei. For every case considered, we define the following:

$$\begin{gathered}
Z \equiv \text{Proton Number} \\
N \equiv \text{Neutron Number} \\
\Delta_p \equiv \text{Proton Pairing Gap} \\
\Delta_n \equiv \text{Neutron Pairing Gap} \\
\text{BE} \equiv \text{Binding Energy} \\
S_p \equiv \text{Proton Separation Energy} \\
S_{2p} \equiv \text{Two-Proton Separation Energy} \\
S_n \equiv \text{Neutron Separation Energy} \\
S_{2n} \equiv \text{Two-Neutron Separation Energy} \\
Q_\alpha \equiv \text{Q Value for } \alpha \text{ Decay}
\end{gathered}$$

Here we note the following:

- N and Z are always defined to be even
- BE is always defined to be negative.
- If $S_{p,n} > 0$, the nucleus is stable to nucleon emission.
- If $S_{2p,2n} > 0$, the nucleus is stable to two-nucleon emission.
- If $Q_\alpha < 0$, the nucleus is stable against α-decay.

For even-even nuclei:

$$S_p\,(Z,N) = \text{BE}\,(Z-1,N) - \text{BE}\,(Z,N) \tag{1}$$

$$S_{2p}\,(Z,N) = \text{BE}\,(Z-2,N) - \text{BE}\,(Z,N) \tag{2}$$

$$S_n\,(Z,N) = \text{BE}\,(Z,N-1) - \text{BE}\,(Z,N) \tag{3}$$

$$S_{2n}\,(Z,N) = \text{BE}\,(Z,N-2) - \text{BE}\,(Z,N) \tag{4}$$

$$Q_\alpha\,(Z,N) = 28.3 + \text{BE}\,(Z,N) - \text{BE}\,(Z-2,N-2) \tag{5}$$

We have every term above except for $\text{BE}\,(Z-1,N)$ and $\text{BE}\,(Z,N-1)$ from equations 1 and 3. In order to calculate them we do the following:

$$\begin{aligned}\mathrm{BE}(Z-1,N) = \frac{1}{2}\Big[\mathrm{BE}(Z,N) + \mathrm{BE}(Z-2,N)\Big] \\ +\frac{1}{2}\Big[\Delta_p(Z,N) + \Delta_p(Z-2,N)\Big]\end{aligned} \tag{6}$$

$$\begin{aligned}\mathrm{BE}(Z,N-1) = \frac{1}{2}\Big[\mathrm{BE}(Z,N) + \mathrm{BE}(Z,N-2)\Big] \\ +\frac{1}{2}\Big[\Delta_n(Z,N) + \Delta_n(Z,N-2)\Big]\end{aligned} \tag{7}$$

For odd-A nuclei (odd neutron number):

$$S_p\,(Z,N-1) = \mathrm{BE}\,(Z-1,N-1) - \mathrm{BE}\,(Z,N-1) \tag{8}$$

$$S_{2p}\,(Z,N-1) = \mathrm{BE}\,(Z-2,N-1) - \mathrm{BE}\,(Z,N-1) \tag{9}$$

$$S_n\,(Z,N-1) = \mathrm{BE}\,(Z,N-2) - \mathrm{BE}\,(Z,N-1) \tag{10}$$

$$S_{2n}\,(Z,N-1) = \mathrm{BE}\,(Z,N-3) - \mathrm{BE}\,(Z,N-1) \tag{11}$$

$$Q_\alpha\,(Z,N-1) = 28.3 + \mathrm{BE}\,(Z,N-1) - \mathrm{BE}\,(Z-2,N-3) \tag{12}$$

We have every term above except for $\mathrm{BE}\,(Z,N-1)$, $\mathrm{BE}\,(Z-1,N-1)$, $\mathrm{BE}\,(Z-2,N-1)$, $\mathrm{BE}\,(Z,N-3)$, and $\mathrm{BE}\,(Z-2,N-3)$.

$\mathrm{BE}\,(Z,N-1)$ is defined in equation 7 above; the others are calculated as follows:

$$\text{BE}(Z-1,N-1) = \frac{1}{2}\Big[\text{BE}(Z,N-1) + \text{BE}(Z-2,N-1)\Big] + \frac{1}{2}\Big[\Delta_p(Z,N-1) + \Delta_p(Z-2,N-1)\Big] \tag{13}$$

$$\text{BE}(Z-2,N-1) = \frac{1}{2}\Big[\text{BE}(Z-2,N) + \text{BE}(Z-2,N-2)\Big] + \frac{1}{2}\Big[\Delta_n(Z-2,N) + \Delta_n(Z-2,N-2)\Big] \tag{14}$$

$$\text{BE}(Z,N-3) = \frac{1}{2}\Big[\text{BE}(Z,N-2) + \text{BE}(Z,N-4)\Big] + \frac{1}{2}\Big[\Delta_n(Z,N-2) + \Delta_n(Z,N-4)\Big] \tag{15}$$

$$\text{BE}(Z-2,N-3) = \frac{1}{2}\Big[\text{BE}(Z-2,N-2) + \text{BE}(Z-2,N-4)\Big] + \frac{1}{2}\Big[\Delta_n(Z-2,N-2) + \Delta_n(Z-2,N-4)\Big] \tag{16}$$

For the proton pairing gaps of equation 13:

$$\Delta_p(Z,N-1) = \frac{1}{2}\Big[\Delta_p(Z,N) + \Delta_p(Z,N-2)\Big] \tag{17}$$

$$\Delta_p(Z-2,N-1) = \frac{1}{2}\Big[\Delta_p(Z-2,N) + \Delta_p(Z-2,N-2)\Big] \tag{18}$$

For odd-A nuclei (odd proton number):

$$S_p(Z-1,N) = \text{BE}(Z-2,N) - \text{BE}(Z-1,N) \tag{19}$$

$$S_{2p}(Z-1,N) = \text{BE}(Z-3,N) - \text{BE}(Z-1,N) \tag{20}$$

$$S_n(Z-1,N) = \text{BE}(Z-1,N-1) - \text{BE}(Z-1,N) \tag{21}$$

$$S_{2n}(Z-1,N) = \text{BE}(Z-1,N-2) - \text{BE}(Z-1,N) \tag{22}$$

$$Q_\alpha(Z-1,N) = 28.3 + \text{BE}(Z-1,N) - \text{BE}(Z-3,N-2) \tag{23}$$

We have every term above except for $\text{BE}(Z-1,N)$, $\text{BE}(Z-3,N)$, $\text{BE}(Z-1,N-1)$, $\text{BE}(Z-1,N-2)$, $\text{BE}(Z-3,N-2)$.

$\text{BE}(Z-1,N)$ and $\text{BE}(Z-1,N-1)$ are defined in equations 6 and 13 respectively; the others are calculated as follows:

$$\mathrm{BE}\left(Z-3,N\right)=\frac{1}{2}\Bigg[\mathrm{BE}\left(Z-2,N\right)+\mathrm{BE}\left(Z-4,N\right)\Bigg]$$
$$+\frac{1}{2}\Bigg[\Delta_{p}\left(Z-2,N\right)+\Delta_{p}\left(Z-4,N\right)\Bigg] \tag{24}$$

$$\mathrm{BE}\left(Z-1,N-2\right)=\frac{1}{2}\Bigg[\mathrm{BE}\left(Z,N-2\right)+\mathrm{BE}\left(Z-2,N-2\right)\Bigg]$$
$$+\frac{1}{2}\Bigg[\Delta_{p}\left(Z,N-2\right)+\Delta_{p}\left(Z-2,N-2\right)\Bigg] \tag{25}$$

$$\mathrm{BE}\left(Z-3,N-2\right)=\frac{1}{2}\Bigg[\mathrm{BE}\left(Z-2,N-2\right)+\mathrm{BE}\left(Z-4,N-2\right)\Bigg]$$
$$+\frac{1}{2}\Bigg[\Delta_{p}\left(Z-2,N-2\right)+\Delta_{p}\left(Z-4,N-2\right)\Bigg] \tag{26}$$

For odd-odd nuclei:

$$S_{p}\left(Z-1,N-1\right)=\mathrm{BE}\left(Z-2,N-1\right)-\mathrm{BE}\left(Z-1,N-1\right) \tag{27}$$

$$S_{2p}\left(Z-1,N-1\right)=\mathrm{BE}\left(Z-3,N-1\right)-\mathrm{BE}\left(Z-1,N-1\right) \tag{28}$$

$$S_{n}\left(Z-1,N-1\right)=\mathrm{BE}\left(Z-1,N-2\right)-\mathrm{BE}\left(Z-1,N-1\right) \tag{29}$$

$$S_{2n}\left(Z-1,N-1\right)=\mathrm{BE}\left(Z-1,N-3\right)-\mathrm{BE}\left(Z-1,N-1\right) \tag{30}$$

$$Q_{\alpha}\left(Z-1,N-1\right)=28.3+\mathrm{BE}\left(Z-1,N-1\right)-\mathrm{BE}\left(Z-3,N-3\right) \tag{31}$$

We have every term except $\mathrm{BE}\left(Z-2,N-1\right)$, $\mathrm{BE}\left(Z-3,N-1\right)$, $\mathrm{BE}\left(Z-1,N-2\right)$, $\mathrm{BE}\left(Z-1,N-3\right)$, and $\mathrm{BE}\left(Z-3,N-3\right)$.

$\mathrm{BE}\left(Z-2,N-1\right)$ and $\mathrm{BE}\left(Z-1,N-2\right)$ are defined in equations 14 and 25 above, respectively; the others are defined as follows:

$$\mathrm{BE}\left(Z-3,N-1\right)=\frac{1}{2}\Big[\mathrm{BE}\left(Z-2,N-1\right)+\mathrm{BE}\left(Z-4,N-1\right)\Big]+\frac{1}{2}\Big[\Delta_p\left(Z-2,N-1\right)+\Delta_p\left(Z-4,N-1\right)\Big] \tag{32}$$

$$\mathrm{BE}\left(Z-1,N-3\right)=\frac{1}{2}\Big[\mathrm{BE}\left(Z,N-3\right)+\mathrm{BE}\left(Z-2,N-3\right)\Big]+\frac{1}{2}\Big[\Delta_p\left(Z,N-3\right)+\Delta_p\left(Z-2,N-3\right)\Big] \tag{33}$$

$$\mathrm{BE}\left(Z-3,N-3\right)=\frac{1}{2}\Big[\mathrm{BE}\left(Z-2,N-3\right)\mathrm{BE}\left(Z-4,N-3\right)\Big]+\frac{1}{2}\Big[\Delta_p\left(Z-2,N-3\right)+\Delta_p\left(Z-4,N-3\right)\Big] \tag{34}$$

For the proton pairing gaps of equations 32, 33, and 34:

$$\Delta_p\left(Z-4,N-1\right)=\frac{1}{2}\Big[\Delta_p\left(Z-4,N\right)+\Delta_p\left(Z-4,N-2\right)\Big] \tag{35}$$

$$\Delta_p\left(Z,N-3\right)=\frac{1}{2}\Big[\Delta_p\left(Z,N-2\right)+\Delta_p\left(Z,N-4\right)\Big] \tag{36}$$

$$\Delta_p\left(Z-2,N-3\right)=\frac{1}{2}\Big[\Delta_p\left(Z-2,N-2\right)+\Delta_p\left(Z-2,N-4\right)\Big] \tag{37}$$

$$\Delta_p\left(Z-4,N-3\right)=\frac{1}{2}\Big[\Delta_p\left(Z-4,N-2\right)+\Delta_p\left(Z-4,N-4\right)\Big] \tag{38}$$